FRQ 4 Practice Focused on Exponential and Log Functions

Practice FRQ 4 Problem A

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 5(2)^{x-4}$$
$$h(x) = 2\log_3(x+1)$$

i. Solve $g(x) = 30$ for all values of x in the domain of g.
ii. Solve $h(x) = 4$ for all values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \ln x - 5\ln 2$$
$$k(x) = 3 - 6\log_2(x) + \log_2(10x^8)$$

i. Rewrite $j(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln$(expression).
ii. Rewrite $k(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2$(expression).

(C) The function m is given by $m(x) = 3^{2x} - 3^x - 20$. Find all input values in the domain of m that yield an output value of 0.

Practice FRQ 4 Problem B

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{3x} - 4$$
$$h(x) = \log_5(2x)$$

i. Solve $g(x) = 8$ for all values of x in the domain of g.

ii. Solve $h(x) = -1$ for all values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = 2\ln(x-4) - \ln(x+3)$$
$$k(x) = -\log_5(3x^5) + 2\log_5(2x^4)$$

i. Rewrite $j(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln$(expression).

ii. Rewrite $k(x)$ as a single logarithm base 5 without negative exponents in any part of the expression. Your result should be of the form $\log_5$(expression).

(C) The function m is given by $m(x) = \log_2(x^2 - 2x + 5)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem C

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \ln(2x) - 5$$
$$h(x) = \frac{(5)^{3x}}{\sqrt{5}}$$

i. Solve $g(x) = 10$ for all values of x in the domain of g.

ii. Solve $h(x) = 25$ for all values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = 2\log(5x^5) - \log(x)$$
$$k(x) = 4\log_2(x+1) - 3 - 2\log_2(x-4)$$

i. Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log(\text{expression})$.

ii. Rewrite $k(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

(C) The function m is given by $m(x) = \log_2(x^2 + 2x - 15) - \log_2(x)$. Find all input values in the domain of m that yield an output value of 2.

Practice FRQ 4 Problem D

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 10e^{(x-4)}$$
$$h(x) = \log_3(2 - x) - \log_3 5$$

i. Solve $g(x) = 10$ for all values of x in the domain of g.
ii. Solve $h(x) = 2$ for all values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_3(6x^3) - \log_3 10 - \log_3(5x^6)$$
$$k(x) = -2 + 2\log_4(x + 1)$$

i. Rewrite $j(x)$ as a single logarithm base 3 without negative exponents in any part of the expression. Your result should be of the form $\log_3$(expression).
ii. Rewrite $k(x)$ as a single logarithm base 4 without negative exponents in any part of the expression. Your result should be of the form $\log_4$(expression).

(C) The function m is given by $m(x) = 2(\log_4 x)^2 - (\log_4 x)$. Find all input values in the domain of m that yield an output value of 0.

Answers to FRQ 4 Practice Focused on Exponential and Log Functions

On the AP Exam, work must be shown where applicable.

Practice FRQ 4 Problem A

(A)

i. $x = 4 + \log_2 6$

ii. $x = 8$

(B)

i. $\ln\left(\frac{x}{32}\right)$

ii. $\log_2(80x^2)$

(C) $x = \log_3 5$

Practice FRQ 4 Problem B

(A)

i. $x = \frac{1}{3}\ln(12)$

ii. $x = \frac{1}{10}$

(B)

i. $\ln\left(\frac{(x-4)^2}{x+3}\right)$

ii. $\log_5\left(\frac{4x^3}{3}\right)$

(C) $x = -1$, $x = 3$

Practice FRQ 4 Problem C

(A)

i. $x = \frac{1}{2}e^{15}$

ii. $x = \frac{5}{6}$

(B)

i. $\log(25x^9)$

ii. $\log_2\left(\frac{(x+1)^4}{8(x-4)^2}\right)$

(C) $x = 5$

Practice FRQ 4 Problem D

(A)

i. $x = 4$

ii. $x = -43$

(B)

i. $\log_3\left(\frac{3}{25x^3}\right)$

ii. $\log_4\left(\frac{(x+1)^2}{16}\right)$

(C) $x = 1$, $x = 2$